8

Annuities and loan repayments

Syllabus reference

Financial mathematics 5

- Annuities and loan repayments

Superannuation (otherwise known as super) is a way of saving money. It is a type of annuity — a regular payment into an investment. This chapter will focus on annuities, make calculations about their future worth and investigate applications of annuities.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching **SkillSHEET**. Either click on the **SkillSHEET** icon next to the question on the *Maths Quest HSC Course* eBookPLUS or ask your teacher for a copy.

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Finding values of *n* and *r* in financial formulas

Finding values of *n* and *r* in financial formulas

1 Find the value of n and r in for each of the following investments.
- **a** Interest of 8% p.a. for 5 years, with interest calculated annually
- **b** Interest of 6% p.a. for 4 years, with interest calculated six-monthly
- **c** Interest of 7.6% p.a. for 3 years, with interest calculated quarterly
- **d** Interest of 9.6% p.a. for 10 years, with interest calculated monthly
- **e** Interest of 24% p.a. for November, with interest calculated daily

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Calculating simple interest

Calculating simple interest

2 Find the simple interest on each of the following investments.
- **a** $25 000 invested at 5% p.a. for 4 years
- **b** $15 500 invested at 8.2% p.a. for 6 years
- **c** $42 000 invested at 9.4% p.a. for 18 months

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Calculating compound interes

Calculating compound interest

3 Find the compound interest earned on each of the following investments.
- **a** $12 000 invested at 6% p.a. for 3 years, with interest compounded annually
- **b** $35 000 invested at 8% p.a. for 5 years, with interest compounded six-monthly
- **c** $56 000 invested at 7.2% p.a. for 4 years, with interest compounded quarterly

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Reading financial tables

Reading financial tables

4 The table below shows the amount to which $1 will grow under compound interest.

	Interest rate per period			
Periods	**6%**	**7%**	**8%**	**9%**
1	1.060	1.070	1.080	1.090
2	1.123	1.145	1.166	1.188
3	1.191	1.225	1.260	1.295
4	1.262	1.311	1.360	1.412

Use the table to find the future value of each of the following investments.
- **a** $8000 at 6% for 2 years, with interest compounded annually
- **b** $12 500 at 8% p.a. for 3 years, with interest compounded annually
- **c** $18 000 at 12% p.a. for 2 years, with interest compounded six-monthly

8A Future value of an annuity

An **annuity** is a form of investment involving regular periodic contributions to an account. On such an investment, interest compounds at the end of each period and the next contribution to the account is then made.

Superannuation is a common example of an annuity. Here, people invest in a fund on a regular basis, the interest on the investment compounds, while the principal is added to for each period. The annuity is usually set aside for a person's entire working life and is used to fund retirement. It may also be used to fund a long-term goal, such as a trip in 10 years' time.

To understand the growth of an annuity, we need to revise compound interest. The compound interest formula is:

$$A = P(1 + r)^n$$

where A is the final balance, P is the initial quantity, r is the interest rate per compounding period and n is the number of compounding periods.

WORKED EXAMPLE 1

Calculate the value of a $5000 investment made at 8% p.a. for 4 years.

THINK	WRITE
1 Write the values of P, r and n.	$P = \$5000, r = 0.08, n = 4$
2 Write the formula.	$A = P(1 + r)^n$
3 Substitute values for P, r and n.	$= \$5000 \times (1.08)^4$
4 Calculate the value of A.	$= \$6802.44$

An annuity takes the form of a sum of compound interest investments. Consider the case of a person who invests $1000 at 10% p.a. at *the end* of each year for five years.

To calculate this, we would need to calculate the value of the first $1000 that is invested for four years, the second $1000 that is invested for three years, the third $1000 that is invested for two years, the fourth $1000 that is invested for one year and the last $1000 that is added to the investment.

WORKED EXAMPLE 2

Calculate the value of an annuity in which $1000 is invested at the end of each year at 10% p.a. for 5 years.

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Worked example 2

THINK	WRITE
1 Use the compound interest formula to calculate the amount to which the first $1000 will grow.	$A = P(1 + r)^n$ $= \$1000 \times 1.1^4$ $= \$1464.10$
2 Use the compound interest formula to calculate the amount to which the second $1000 will grow.	$A = P(1 + r)^n$ $= \$1000 \times 1.1^3$ $= \$1331.00$
3 Use the compound interest formula to calculate the amount to which the third $1000 will grow.	$A = P(1 + r)^n$ $= \$1000 \times 1.1^2$ $= \$1210.00$
4 Use the compound interest formula to calculate the amount to which the fourth $1000 will grow.	$A = P(1 + r)^n$ $= \$1000 \times 1.1$ $= \$1100.00$
5 Find the total of the separate $1000 investments, remembering to add the final $1000.	Total value $= \$1464.10 + \$1331.00 + \$1210.00 + \$1100.00 + \$1000$ $= \$6105.10$

In most cases it is more practical to calculate the total value of an annuity using a formula. The amount to which an annuity grows is called the **future value of an annuity** and can be calculated using the formula:

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

where M is the contribution per period paid at the end of the period, r is the interest rate per period expressed as a decimal, and n is the number of deposits.

For the above example: $A = M\left\{\frac{(1+r)^n - 1}{r}\right\} = \$1000\left\{\frac{1.1^5 - 1}{0.1}\right\} = \$6105.10$

WORKED EXAMPLE 3

Bernie invests $2000 in a retirement fund at 5% p.a. interest compounded annually at the end of each year for 20 years. Calculate the future value of this annuity at retirement.

THINK	WRITE
1 Write the values of M, r and n.	$M = \$2000,\ r = 0.05,\ n = 20$
2 Write the formula.	$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$
3 Substitute values for M, r and n.	$= \$2000\left\{\frac{1.05^{20} - 1}{0.05}\right\}$
4 Calculate.	$= \$66\,131.91$

In some examples, calculations will need to be made when contributions are made more often than once a year and when interest compounds more often than once a year.

WORKED EXAMPLE 4

Christina invests $500 in a fund every 6 months at 9% p.a. interest, compounding six-monthly for 10 years. Calculate the future value of the annuity after 10 years.

THINK	WRITE
1 Write the values of M, r and n by considering the interest rate as 4.5% per interest period and 20 interest periods.	9% p.a. = 4.5% for 6 months So, $r = 0.045$ and $n = 20$.
2 Write the formula.	$A = M\left\{\dfrac{(1+r)^n - 1}{r}\right\}$
3 Substitute for M, r and n.	$= \$500\left\{\dfrac{1.045^{20} - 1}{0.045}\right\}$
4 Calculate.	$= \$15\,685.71$

If we rearrange the formula for an annuity to make M (the contribution per period) the subject of the formula, we have:

$$M = \frac{Ar}{(1+r)^n - 1}$$

This formula would be used when we know the final amount to be saved and wish to calculate the amount of each regular deposit.

WORKED EXAMPLE 5

Vikki has the goal of saving $10 000 in the next five years. The best interest rate that she can obtain is 8% p.a., with interest compounded annually. Calculate the amount of each annual contribution that Vikki must make.

THINK	WRITE
1 Write the values of A, r and n.	$A = \$10\,000$, $r = 0.08$, $n = 5$
2 Write the formula.	$M = \dfrac{Ar}{(1+r)^n - 1}$
3 Substitute for A, r and n. *Hint:* insert brackets when using your calculator.	$= \dfrac{(10\,000 \times 0.08)}{(1.08^5 - 1)}$
4 Calculate the value of M.	$= \$1704.56$

REMEMBER

1. The compound interest formula is:
$$A = P(1 + r)^n$$
where A is the final balance, r is the interest rate per period expressed as a decimal and n is the number of compounding periods.
2. An annuity is a form of investment where periodical equal contributions are made to an account, with interest compounding at the end of each period.

3. The value of an annuity is calculated by adding the value of each amount contributed as a separate compound interest investment.
4. We can calculate the value of an annuity by using the formula:

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

where M is the contribution per period, paid at the end of the period, r is the interest rate per period expressed as a decimal and n is the number of deposits.

5. The amount of each contribution to annuity to reach a certain goal can be calculated using the formula:

$$M = \frac{Ar}{(1+r)^n - 1}$$

EXERCISE

8A Future value of an annuity

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Finding values of *n* and *r* in financial formulas

1 WE1 Calculate the value after 5 years of an investment of $4000 at 12% p.a., with interest compounded annually.

2 Calculate the value to which each of the following compound interest investments will grow.

- **a** $5000 at 6% p.a. for 5 years, with interest calculated annually
- **b** $12 000 at 12% p.a. for 3 years, with interest calculated annually
- **c** $4500 at 8% p.a. for 4 years, with interest compounded six-monthly
- **d** $3000 at 9.6% p.a. for 3 years, with interest compounded six-monthly
- **e** $15 000 at 8.4% p.a. for 2 years, with interest compounded quarterly
- **f** $2950 at 6% p.a. for 3 years, with interest compounded monthly

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Calculating simple interest

3 WE2 At the end of each year for four years Rodney invests $1000 in an investment fund that pays 7.5% p.a. interest, compounded annually. By calculating each investment of $1000 separately, use the compound interest formula to calculate the future value of Rodney's investment after four years.

4 Caitlin is saving for a holiday in two years and so every six months she invests $2000 in an account that pays 7% p.a. interest, with the interest compounding every six months.

- **a** Use the compound interest formula to calculate the amount to which the:
  - **i** first investment of $2000 will grow
  - **ii** second investment of $2000 will grow
  - **iii** third investment of $2000 will grow
  - **iv** fourth investment of $2000 will grow.
- **b** If Caitlin then adds a final deposit of $2000 to her account immediately before her holiday, what is the total value of her annuity?

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Calculating compound interest

5 WE3 Use the formula $A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$ to find the future value of an annuity in which $1000 is invested each year for 25 years at an interest rate of 8% p.a.

6 When baby Shannon was born, her grandparents deposited $500 in an account that pays 6% p.a. interest, compounded annually. They added $500 to the account each birthday, making the last deposit on Shannon's 21st birthday.

- **a** How many deposits of $500 were made?
- **b** The investment was given to Shannon as a 21st birthday present. What was the total value of the investment at this point? (*Hint:* Use the answer to part **a**.)
- **c** Shannon's grandparents advised Shannon to keep adding $500 to the investment each birthday so that she had a retirement fund at age 60. If Shannon follows this advice, what will the investment be worth at age 60? (Assume Shannon makes the last deposit on her 60th birthday.)

7 Calculate the future value of each of the following annuities.
- **a** $2000 invested at the end of each year for 10 years, at an interest rate of 5% p.a.
- **b** $5000 invested at the end of each year for 5 years, at an interest rate of 8% p.a.
- **c** $10 000 invested at the end of each year for 20 years, at an interest rate of 7.5% p.a.
- **d** $500 invested at the end of each year for 30 years, at an interest rate of 15% p.a.
- **e** $25 000 invested at the end of each year for 4 years, at an interest rate of 9.2% p.a.

8 Darlene is saving for a deposit on a unit. She hopes to buy one in four years and needs a $30 000 deposit, so she invests $5000 per year in an annuity at 7.5% p.a. starting on 1 January 2007.
- **a** After the last deposit is made on 1 January 2011, how many deposits has Darlene made?
- **b** Use the annuity formula to calculate if Darlene would have saved enough for her deposit.
- **c** How much interest was paid to Darlene on this annuity?

9 WE4 At the end of every six months Jason invests $800 in a retirement fund which pays interest at 6% p.a., with interest compounded six-monthly. Jason does this for 25 years. Calculate the future value of Jason's annuity after 25 years.

10 Calculate the future value of each of the following annuities on maturity.
- **a** $400 invested at the end of every six months for 12 years at 12% p.a., with interest compounded six-monthly
- **b** $1000 invested at the end of every quarter for 5 years at 8% p.a., with interest compounded every quarter
- **c** $2500 invested at the end of each quarter at 7.2% p.a. for 4 years, with interest compounded quarterly
- **d** $1000 invested at the end of every month for 5 years at 6% p.a., with interest compounded monthly

11 MC The interest earned on $10 000 invested at 8% p.a. for 10 years, with interest compounded annually, is:

A $11 589.25 **B** $21 589.25
C $134 865.62 **D** $144 865.62

12 MC Tracey invests $500 in a fund at the end of each year for 20 years. The fund pays 12% p.a. interest, compounded annually. The total amount of interest that Tracey earns on this fund investment is:

A $4323.15 **B** $4823.23
C $26 026.22 **D** $36 026.22

13 WE5 Thomas has the goal of saving $400 000 for his retirement in 25 years. If the best interest rate that Thomas can obtain is 10% p.a., with interest compounded annually, calculate the amount of each annual contribution that Thomas will need to make.

14 Calculate the amount of each annual contribution needed to obtain each of the following amounts.
- **a** $25 000 in 5 years at 5% p.a., with interest compounded annually
- **b** $100 000 in 10 years at 7.5% p.a., with interest compounded annually
- **c** $500 000 in 40 years at 8% p.a., with interest compounded annually

15 Leanne is 24 years old and invests $30 per week in her superannuation fund. Leanne's employer matches this amount.
- **a** If Leanne plans to retire at 60, calculate the total that Leanne will contribute to the fund at this rate.
- **b** Calculate the total contributions that will be made to the fund at this rate.
- **c** If the fund returns an average 4% p.a. interest, compounded annually, calculate the future value of Leanne's superannuation.

16 **a** Find the amount of each annual contribution needed to achieve a future value of $100 000 if the investment is made for 10 years at an interest rate of 11% p.a., with interest compounding annually.
b Find the amount of each quarterly contribution needed to save $15 000 in five years at 12% p.a., with interest compounding quarterly.
c Find the amount of each six-monthly contribution to an annuity if the savings goal is $50 000 in 15 years and the interest rate is 8% p.a., with interest compounding six-monthly.

Further development

17 Simon invests $2000 per year into an annuity that pays 8% p.a. over a 10-year period.
a Calculate the future value of Simon's annuity given that interest is compounded annually.
b Calculate the future value of the annuity if Simon were to make six-monthly deposits of $1000 and interest were to be compounded six-monthly.
c Calculate the percentage increase in the future value of the annuity if payments are made and interest compounded six-monthly. Give your answer correct to 1 decimal place.

18 Stephanie invests $1200 per year into an annuity at 6% p.a. with interest compounded annually.
a If Stephanie plans to invest each year for 20 years, find the future value of the annuity.
b After 10 years Stephanie finds that she is able to increase her annual payment to $1500. By considering the extra $300 as a separate annuity for the second 10 years, find the total value of the annuity after 20 years.

19 Archie invests $2000 per year into an annuity. The annuity pays 6.5% p.a. interest with interest compounded annually.
a Calculate the future value of the annuity after 10 years.
b At the conclusion of 10 years Archie is able to invest at the higher rate of 7.2% p.a. with interest compounded annually. Archie takes his annuity and invests the entire sum for a further 10 years at the higher rate. Calculate the value of this investment at maturity.
c At the same time Archie continues to put $2000 per year into the annuity at the higher rate. Calculate the future value of this annuity after a further 10 years.
d Calculate the total value of Archie's annuity at the conclusion of 20 years.

20 Jodie invests $200 per month into her superannuation fund. The interest rate is 6% p.a. with interest compounded monthly.
a Calculate the future value of Jodie's investment for her working life of 40 years.
b Calculate the future value of the investment if her employer matches Jodie's contribution dollar for dollar.

21 Jeff earns an annual salary of $57 500. Jeff contributes $57 per week into his superannuation fund and his employer makes a contribution of 3% of his salary.
a Calculate the total weekly contribution to Jeff's superannuation fund.
b If the superannuation fund earns an average 7.8% p.a. interest compounded weekly, calculate the value of Jeff's investment at retirement after 45 years of working.

22 Tony invests $1000 per year into an annuity paying 9% p.a. interest compounded annually.
a Calculate the future value of Tony's annuity after 5 years.
b For the second payment Tony increases his payment to $1200. By considering the extra $200 as a separate annuity that runs for 4 years, calculate the amount by which the future value will increase.
c Tony increases each payment by $200 for the full five years. Calculate the value of his investment after 5 years.

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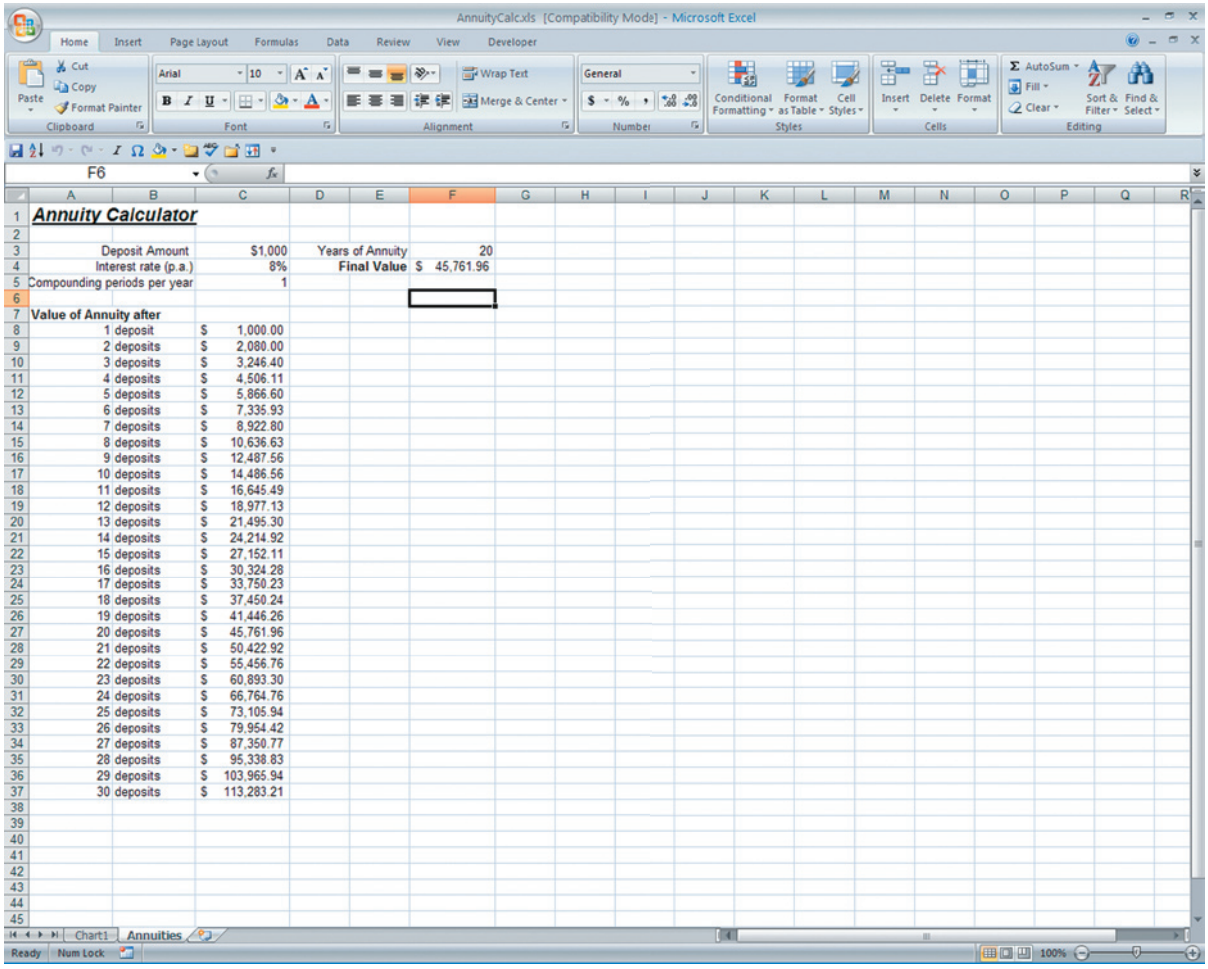
Annuity calculator

Computer Application 1: Annuity calculator

Access the spreadsheet 'Annuity calculator' from the *Maths Quest General Mathematics HSC Course* eBookPLUS. The spreadsheet will show you the growth of an annuity in which $1000 is invested at the end of each year for 20 years at a rate of 8% p.a. interest, compounding annually.



1. The spreadsheet shows that after 20 years the value of this investment is $45 761.96. Below is the growth of the annuity after each deposit is made. This will allow you to see the growth for up to 30 deposits. From the **Edit** menu, use the **Fill Down** functions on the spreadsheet to see further.
2. Click on the tab, 'Chart 1'. This is a line graph that shows the growth of the annuity for up to 30 deposits.
3. Change the size of the deposit to $500 and the compounding periods to 2. This will show how much benefit can be achieved by reducing the compounding period.
4. Check your answers to the previous exercise by using the spreadsheet.

8B Present value of an annuity

To compare an annuity with a single sum investment, we need to use the present value of the annuity. The **present value of an annuity** is the single sum of money that, invested on the same terms as the annuity, will produce the same financial result.

To calculate the present value of an annuity, N, we can use the formula:

$$N = \frac{A}{(1+r)^n}$$

where A is the future value of the annuity
r is the percentage interest rate per compounding period, expressed as a decimal
n is the number of deposits to be made in the annuity.

WORKED EXAMPLE 6

Ashan has an annuity that has a future value of $500 000 on his retirement in 23 years. The annuity is invested at 8% p.a., with interest compounded annually. Calculate the present value of Ashan's annuity.

THINK	WRITE
1 Write the values of A, r and n.	$A = \$500\,000,\ r = 1.08,\ n = 23$
2 Write the formula.	$N = \frac{A}{(1+r)^n}$
3 Substitute for A, r and n.	$= \frac{500\,000}{1.08^{23}}$
4 Calculate.	$= \$85\,157.64$

In many cases you will not know the future value of the annuity when calculating the present value. You will know only the amount of each contribution, M. We know that:

$$N = \frac{A}{(1+r)^n}$$

Using the formula $A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$ to substitute for A gives:

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

This formula allows us to calculate the single sum needed to be invested to give the same financial result as an annuity where we are given the size of each contribution.

WORKED EXAMPLE 7

Jenny has an annuity to which she contributes $1000 per year at 6% p.a. interest, compounded annually. The annuity will mature in 25 years. Calculate the present value of the annuity.

THINK	WRITE
1 Write the values of M, r and n.	$M = \$1000,\ r = 0.06,\ n = 25$
2 Write the formula.	$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$
3 Substitute for M, r and n.	$= 1000 \times \left\{\frac{1.06^{25} - 1}{0.06 \times 1.06^{25}}\right\}$
4 Calculate.	$= \$12\,783.36$

This present value formula can be used to compare investments of different types. The investment with the greater present value will produce the greater financial outcome over time.

WORKED EXAMPLE 8

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Worked example 8

Which of the following investments would give the greater financial return?
Investment A: an annuity of $100 deposited per month for 20 years at 12% p.a. interest, compounding six-monthly
Investment B: a single deposit of $10 000 invested for 20 years at 12% p.a., with interest compounding six-monthly

THINK	WRITE
1 The investments can be compared by calculating the present value of the annuity.	
2 Consider the deposits of $100 per month to be $600 every six months.	
3 Write the values of M, r and n.	$M = \$600,\ r = 0.06,\ n = 40$
4 Write the formula.	$N = M\left\{\dfrac{(1+r)^n - 1}{r(1+r)^n}\right\}$
5 Substitute for M, r and n.	$= \$600 \times \left\{\dfrac{1.06^{40} - 1}{0.06 \times 1.06^{40}}\right\}$
6 Calculate.	$= \$9027.78$
7 Make a conclusion.	The annuity has a lower present value than the single investment. Therefore, the investment of $10 000 will produce a greater outcome over 20 years.

REMEMBER

1. The present value of an annuity is the single sum that can be invested under the same terms as an annuity and will produce the same financial outcome.
2. The present value of an annuity can be calculated using the formula:
$$N = \frac{A}{(1+r)^n}$$
when we know the future value of the annuity.
3. If we know the amount of each contribution of the annuity, we can calculate the present value using the formula
$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
where M is the contribution per period, paid at the end of the period
r is the percentage interest rate per compounding period (expressed as a decimal)
n is the number of interest periods.
4. Investments can be compared using the present value formula. The investment with the greater present value will produce the greater financial outcome over time.

EXERCISE

8B Present value of an annuity

1 WE6 Calculate the present value of an investment that is needed to have a future value of $100 000 in 30 years' time if it is invested at 9% p.a., with interest compounded annually.

2 Calculate the present value of an investment required to generate a future value of:
- **a** $20 000 in 5 years' time at 10% p.a., with interest compounded annually
- **b** $5000 in 4 years' time at 7.2% p.a., with interest compounded annually
- **c** $250 000 in 20 years' time at 5% p.a., with interest compounded annually.

3 Calculate the present value of an investment at 7.2% p.a., with interest compounded quarterly, if it is to have a future value of $100 000 in 10 years' time.

4 Calculate the present value of the investment required to produce a future value of $500 000 in 30 years' time at 9% p.a., with interest compounded:
- **a** annually
- **b** six-monthly
- **c** quarterly
- **d** monthly.

5 WE7 Craig is paying into an annuity an amount of $500 per year. The annuity is to run for 10 years and interest is paid at 7% p.a., with interest compounded annually. Calculate the present value of this annuity.

6 Calculate the present values of each of the following annuities.
- **a** $1000 per year for 30 years at 8% p.a., with interest compounded annually
- **b** $600 per year for 20 years at 7.5% p.a., with interest compounded annually
- **c** $4000 per year for 5 years at 11% p.a., with interest compounded annually
- **d** $200 per month for 25 years at 8.4% p.a., with interest compounded annually

7 Darren pays $250 per month into an annuity that pays 5.6% p.a. interest, compounded quarterly. If the annuity is to run for 10 years, calculate the present value of the annuity.

8 Calculate the present value of a 40-year annuity with interest at 9.6% p.a., compounded monthly, if the monthly contribution to the annuity is $50.

9 MC An annuity is at 12% p.a. for 10 years, with interest compounded six-monthly, and has a future value of $100 000. The present value of the annuity is:

A $31 180.47 **B** $32 197.32
C $310 584.82 **D** $320 713.55

10 MC An annuity consists of quarterly deposits of $200 that are invested at 8% p.a., with interest compounded quarterly. The annuity will mature in 23 years. The present value of the annuity is:

A $1236.65 **B** $2074.21 **C** $8296.85 **D** $8382.72

11 WE8 Which of the following investments will have the greater financial outcome?

Investment A: an annuity of $400 per year for 30 years at 6.9% p.a., with interest compounded annually

Investment B: a single investment of $5000 for 30 years at 6.9% p.a., with interest compounded annually

12 MC Which of the following investments will have the greatest financial outcome?
- **A** An annuity of $1200 per year for 30 years at 8% p.a., with interest compounded annually
- **B** An annuity of $600 every six months for 30 years at 7.9% p.a., with interest compounded six-monthly
- **C** An annuity of $300 every quarter for 30 years at 7.8% p.a., with interest compounded quarterly
- **D** An annuity of $100 per month at 7.5% p.a., for 30 years with interest compounded monthly

13 Kylie wants to take a world trip in 5 years' time. She estimates that she will need $25 000 for the trip. The best investment that Kylie can find pays 9.2% p.a. interest, compounded quarterly.

a Calculate the present value of the investment needed to achieve this goal.

b Kylie plans to save for the trip by depositing $100 per week into an annuity. Calculate if this will be enough for Kylie to achieve her savings goal (take 13 weeks = 1 quarter).

Further development

14 An investment is to grow to $250 000 over 25 years at an average interest rate of 6% p.a.

a Calculate the present value of the annuity if interest is compounded annually.

b Calculate the present value if interest is compounded quarterly.

c Explain why the lower present value indicates that the quarterly investment is better.

d Calculate the percentage reduction in the present value for when the interest is compounded quarterly.

15 An annuity of $2400 is paid every year into an investment paying 9% p.a. over a 20-year period.

a Calculate the present value of the annuity if interest is compounded annually.

b Calculate the present value of the annuity if $200 is paid per month and interest is compounded monthly.

c Calculate the percentage change in the present value if interest is compounded monthly.

16 Andrew makes an investment of $1000 each year into an annuity at 8% p.a. for 10 years. John obtains 10% p.a. interest and claims the present value of the annuity will be 25% less. Determine if John is correct.

17 Ricky makes an investment of $300 per quarter into an annuity that pays 6.6% p.a. with interest compounded quarterly. The annuity is to run for 20 years.

a Find the future value of the annuity.

b Calculate the present value of the annuity.

18 Fiona invests $1800 per year for 30 years into an annuity that pays 7.2% p.a. with interest compounded annually.

a Calculate the future value of the annuity.

b Calculate the present value of the annuity.

c Calculate the present value of an equivalent annuity where contributions are made, and interest is compounded, monthly.

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19 An annuity that is invested at 6.9% p.a. over 10 years with interest compounded annually has a present value of $50 000. Calculate the amount of each annual contribution.

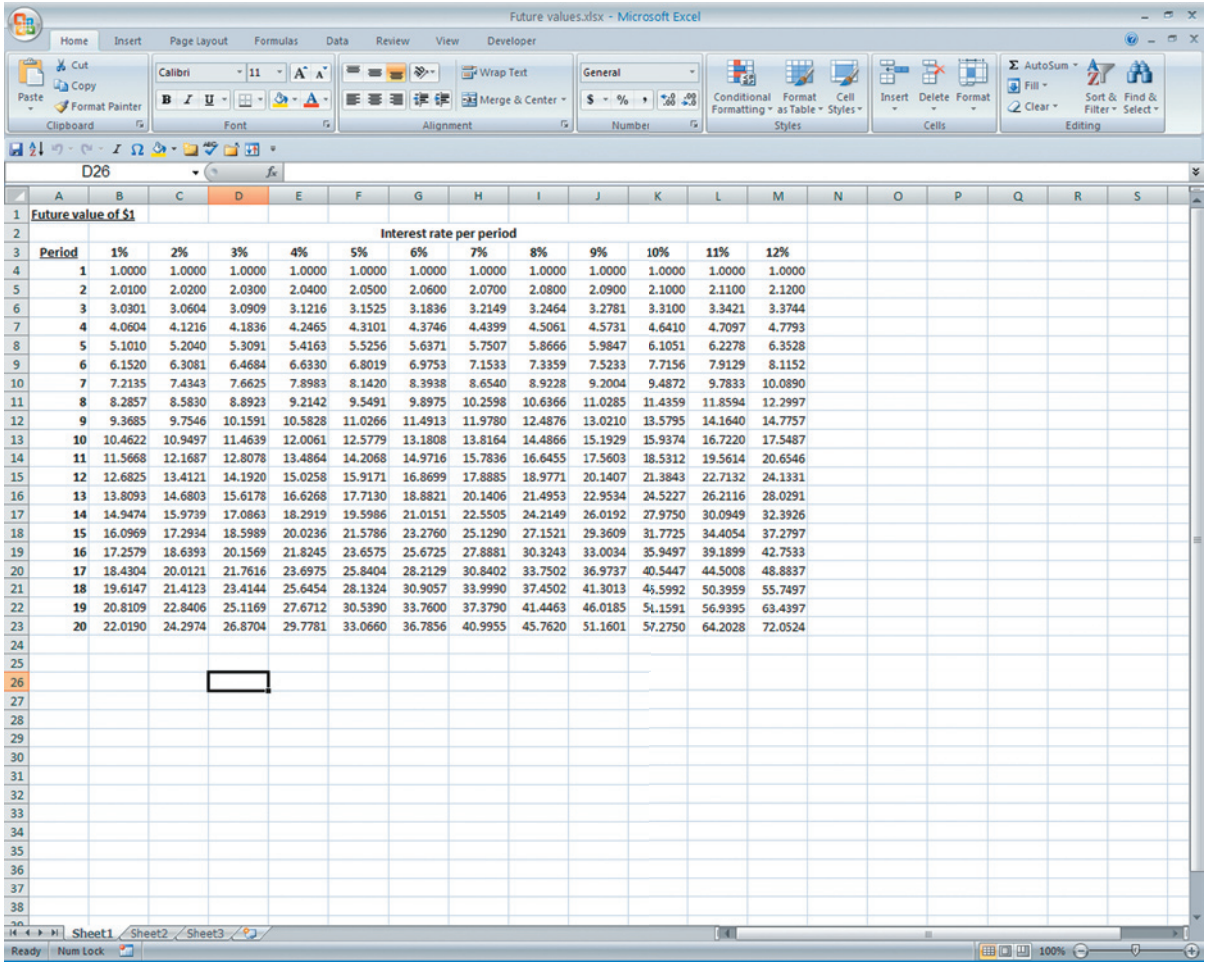
8C Future and present value tables

Problems associated with annuities can be simplified by creating a table that will show either the future value or present value of an annuity of $1 invested per interest period.

Computer Application 2: Future value of $ 1

Consider $1 is invested into an annuity each interest period. The table we are going to construct on a spreadsheet shows the future value of that $1.

1. Open a new spreadsheet.
2. Type in the following information as shown in step 3.
3. In cell **B4** enter the formula **=((1+B$3)^$A4-1)/B$3**. (This is the future value formula from exercise 2A with the value of *M* omitted, as it is equal to 1.) Format the cell, correct to 4 decimal places.
4. Highlight the range of cells **B3** to **M23**. From the **Edit** menu, use **Fill Down** and **Fill Right** functions to copy the formula to all other cells in this range.



Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524

This completes the table. The table shows the future value of an annuity of $1 invested for up to 10 interest periods at up to 10% per interest period. You can extend the spreadsheet further for other interest rates and longer investment periods.

The following table is the set of future values of $1 invested into an annuity. This is the table you should have obtained in computer application 2.

A table such as this can be used to find the value of an annuity by multiplying the amount of the annuity by the future value of $1.

Future value of $1												
	Interest rate per period											
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524

WORKED EXAMPLE 9

Use the table to find the future value of an annuity into which $1500 is deposited at the end of each year at 7% p.a. interest, compounded annually for 9 years.

THINK

1. Look up the future value of $1 at 7% p.a. for 9 years.
2. Multiply this value by 1500.

WRITE

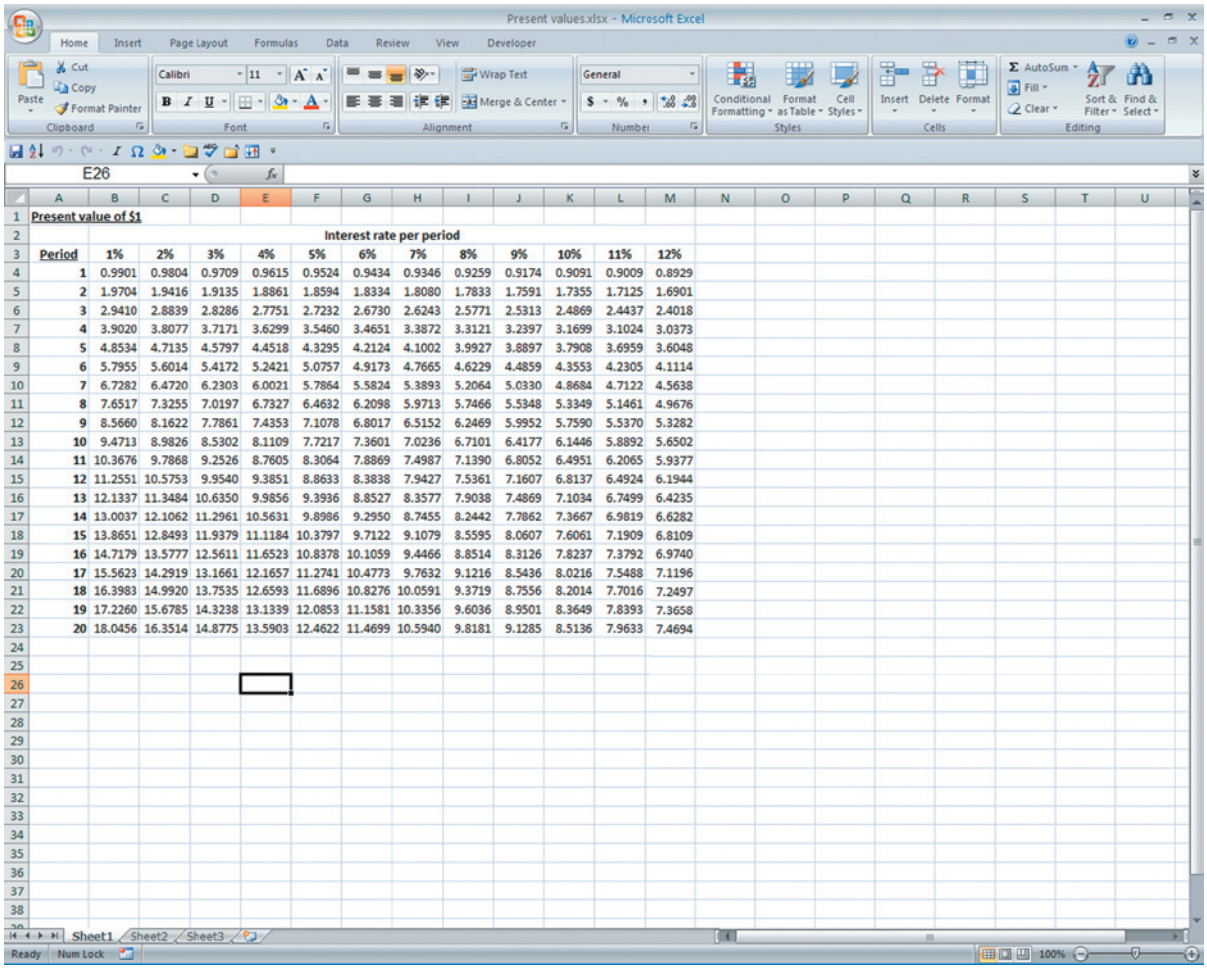
Future value = $1500 × 11.9780
= $17 967

Just as we have a table for the future value of an annuity, we can create a table for the present value of an annuity.

Computer Application 3: Present value table

The table we are about to make on a spreadsheet shows the present value of an annuity of $1 invested per interest period.

1. Open a new spreadsheet.
2. Enter the following information.
3. In cell **B4** type the formula **=((1+B$3)^$A4–1)/(B$3*(1+B$3)^$A4)**.
4. Drag from cell **B4** to **K13**, and then from the **Edit** menu use the **Fill Down** and **Fill Right** functions to copy this formula to the remaining cells in your table.



The table created in computer application 3 shows the present value of an annuity of $1 per interest period for up to 10% per interest period and for up to 10 interest periods.

The table that you have generated is shown below.

	Present value of $1											
	Interest rate per period											
Period	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**11%**	**12%**
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377

(continued)

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.7016	7.2497
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.8393	7.3658
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.9633	7.4694

This table can be used in the same way as the future values table.

WORKED EXAMPLE 10

Liam invests $750 per year in an annuity at 6% per annum for 8 years, with interest compounded annually. Use the table to calculate the present value of Liam's annuity.

THINK	WRITE
1 Use the table to find the present value of a $1 annuity at 6% for 8 interest periods.	
2 Multiply this value by 750.	Present value = $750 × 6.2098 = $4657.35

REMEMBER

1. A table of future values shows the future value of an annuity in which $1 is invested per interest period.
2. A table of present values shows the present value of an annuity in which $1 is invested per interest period.
3. A table of present or future values can be used to compare investments and determine which will give the greater financial return.

EXERCISE 8C

Future and present value tables

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Reading financial tables

1 WE9 Use the table of future values on page 251 to determine the future value of an annuity of $800 invested per year for 5 years at 9% p.a., with interest compounded annually.

2 Use the table of future values to determine the future value of each of the following annuities.
- **a** $400 invested per year for 3 years at 10% p.a., with interest compounded annually
- **b** $2250 invested per year for 8 years at 8% p.a., with interest compounded annually
- **c** $625 invested per year for 10 years at 4% p.a., with interest compounded annually
- **d** $7500 invested per year for 7 years at 6% p.a., with interest compounded annually

3 Samantha invests $500 every 6 months for 5 years in an annuity at 8% p.a., with interest compounded every 6 months.
- **a** What is the interest rate per interest period?
- **b** How many interest periods are there in Samantha's annuity?
- **c** Use the table to calculate the future value of Samantha's annuity.

4 Use the table to calculate the future value of each of the following annuities.
- **a** $400 invested every 6 months for 4 years at 14% p.a., with interest compounded six-monthly
- **b** $600 invested every 3 months for 2 years at 12% p.a., with interest compounded quarterly
- **c** $100 invested every month for 5 years at 10% p.a., with interest compounded six-monthly

5 Use the table of future values to determine whether an annuity at 5% p.a. for 6 years or an annuity at 6% p.a. for 5 years will produce the greatest financial outcome. Explain your answer.

6 MC Use the table of future values to determine which of the following annuities will have the greatest financial outcome.
- **A** 6% p.a. for 8 years, with interest compounded annually
- **B** 8% p.a. for 6 years, with interest compounded annually
- **C** 7% p.a. for 7 years, with interest compounded annually
- **D** 10% p.a. for 5 years, with interest compounded six-monthly

7 WE10 Use the table of present values on page 252 to determine the present value of an annuity of $1250 per year for 8 years invested at 9% p.a.

8 Use the table of present values to determine the present value of each of the following annuities.
- **a** $450 per year for 5 years at 7% p.a., with interest compounded annually
- **b** $2000 per year for 10 years at 10% p.a., with interest compounded annually
- **c** $850 per year for 6 years at 4% p.a., with interest compounded annually
- **d** $3000 per year for 8 years at 9% p.a., with interest compounded annually

Further development

9 Barbara currently has $60 000 in an investment account that is averaging an interest rate of 6% p.a. compounded annually. She wants to calculate the amount that she will receive after 20 years.
- **a** Use the compound interest formula to find the value of the $60 000 investment at maturity.
- **b** If she deposits $9000 each year, use the table on page 000 to find the future value of this annuity.
- **c** If she places her $60 000 into a different savings account that can offer 8% p.a. compounded annually and increases her deposits to $10 000 each year, find the amount available to her after 20 years.
- **d** Over the 20-year period, calculate the extra amount saved by investing $60 000 in an investment account and $10 000 each year at 8% p.a. compared with $9000 each year at 6% p.a.

10 Find the final value of the following annuity investments by using the compound interest formula together with the table on page 261.
- **a** An initial amount of $10 000 earning 6% p.a. with annual contributions of $7000 for the next 20 years
- **b** An initial amount of $400 000 earning 10% p.a. with annual contributions of $12 000 for the next 5 years
- **c** An initial amount of $100 000 earning 8% p.a. compounded six-monthly with six-monthly contributions of $1200 for the next 9 years
- **d** An initial amount of $0 earning 12% p.a. compounded quarterly with quarterly contributions of $1200 for the next 5 years

11 Justin is aged 42 and is planning to retire at 60 years of age. He estimates that he needs $680 000 to provide for his retirement. His current superannuation fund has a balance of $40 000 and is delivering 12% p.a. compounded annually.
- **a** Calculate the value of the $40 000 when Justin reaches retirement age.
- **b** Use the table on page 000 to find the annual contributions (correct to the nearest $10) needed to meet the retirement lump-sum target.

12 Johnny has an annuity of $1500 per year at 5.5% p.a. interest compounded annually over a 20-year period.
- **a** Use the table on page 261 to estimate the future value of a $1 annuity over a 20-year period.
- **b** Use your answer to (a) to find the future value of Johnny's annuity.
- **c** Use the table on pages 262–3 to estimate the present value of the annuity.

13 Jodie has an annuity into which she pays $1800 every quarter. The interest rate is 10% p.a. with interest compounded quarterly. The annuity is to run for 4 years.
- **a** Use the table to estimate the future value of the annuity.
- **b** Jodie's fiancé Paul wants to make a single investment with a future value equal to Jodie's annuity. Find the amount of the investment that Paul must make.

14 Find the contributions required to meet the following superannuation goals.
- **a** A final payout of $800 000 with 10 years to go, with a superannuation fund delivering 6% p.a. compounded annually. Contributions are to be paid once a year.
- **b** A final payout of $375 000 with 20 years to go. The contribution is paid monthly into a fund averaging 9% p.a. compounded annually.
- **c** A final payout of $1 million with 20 years to go. The contribution is paid monthly into a fund averaging 12% p.a. compounded annually.

8D Loan repayments

When a loan is taken out and is repaid in equal monthly instalments, the pattern of repayments works similar to an annuity. Each month interest compounds on the balance owing on the loan and then a repayment is made.

Consider a loan where the amount borrowed is equal to the present value of the annuity, N, and the amount paid on the loan each month is equal to the contribution to the annuity per period, M. Use the formula for present value, $N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$. To calculate the amount of each monthly repayment, we need to make M the subject of this formula. When we do this the formula becomes:

$$M = N\left\{\frac{r(1+r)^n}{(1+r)^n - 1}\right\}$$

In this formula, M is the amount of each repayment, N is the amount borrowed, r is the interest rate per repayment period as a decimal and n is the number of repayments to be made.

This formula is not given to you on the formula sheet but will be given to you if it is needed to solve a problem in the exam.

WORKED EXAMPLE 11

Use the formula $M = N\left\{\frac{r(1+r)^n}{(1+r)^n - 1}\right\}$ to calculate the monthly repayments on a loan of $5000 to be repaid in monthly instalments over 4 years at an interest rate of 12% p.a.

THINK	WRITE
1 Calculate the values of r and n.	$r = 0.01$ and $n = 48$
2 Write the formula.	$M = N\left\{\frac{r(1+r)^n}{(1+r)^n - 1}\right\}$

3	Substitute for N, r and n.	$= 5000 \times \left\{ \frac{0.01 \times 1.01^{48}}{1.01^{48} - 1} \right\}$
4	Calculate.	$= \$131.67$

Having worked out the amount of each monthly repayment, we are also able to calculate the total cost of repaying a loan by multiplying the amount of each repayment by the number of repayments.

WORKED EXAMPLE 12

Calculate the total cost of repaying a $100 000 home loan at 9% p.a. in equal monthly repayments over a 25-year term.

THINK	WRITE
1 Calculate the values of r and n.	$r = 0.0075$, $n = 300$
2 Write the formula.	$M = N \left\{ \frac{r(1+r)^n}{(1+r)^n - 1} \right\}$
3 Substitute for N, r and n.	$= 100\,000 \times \left\{ \frac{0.0075 \times 1.0075^{300}}{1.0075^{300} - 1} \right\}$
4 Calculate the amount of each monthly repayment.	$= \$839.20$
5 Calculate the total repayments on the loan.	Total repayments $= \$839.20 \times 300$ $= \$251\,760$

By increasing the amount of each repayment, we are able to shorten the term of the loan. There is no easy method to calculate the amount of time that it will take to repay a loan. To do this we use a 'guess and refine' method. We adjust the value of n in the formula until the amount of the repayment is reached.

WORKED EXAMPLE 13

A $100 000 home loan is taken out over a 25-year term at an interest rate of 12% p.a. reducible interest. The minimum monthly repayment on the loan is $1053.22. How long will it take the loan to be repaid at $1200 per month?

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Worked example 13

THINK	WRITE
1 Calculate the value of r.	$r = 0.01$
2 Write the formula.	$M = N \left\{ \frac{r(1+r)^n}{(1+r)^n - 1} \right\}$
3 Take a guess for the value of n (we will take 200 since for the original loan $n = 300$) and substitute.	If $n = 200$, $M = 100\,000 \times \left\{ \frac{0.01 \times 1.01^{200}}{1.01^{200} - 1} \right\}$
4 Calculate the repayment with $n = 200$. As this is less than $1200 we need to further reduce the value of n.	$= \$1158.33$
5 Substitute into the formula with $n = 150$.	If $n = 150$, $M = 100\,000 \times \left\{ \frac{0.01 \times 1.01^{150}}{1.01^{150} - 1} \right\}$

6	Calculate the repayment. As the result is greater than \$1200, we need to increase the value of n.	$= \$1289.99$
7	Substitute into the formula with $n = 180$.	If $n = 180$, $M = 100\,000 \times \left\{ \frac{0.01 \times 1.01^{180}}{1.01^{180} - 1} \right\}$
8	As this is approximately equal to \$1200, it will take 180 months to repay the loan.	$= \$1200.17$
9	Give a written answer.	It will take 15 years to repay the loan.

REMEMBER

1. By considering the amount borrowed in a loan as the present value of an annuity, we can use the present value formula to calculate the amount of each repayment.
2. The formula used to calculate the amount of each monthly repayment is:
$$M = N\left\{\frac{r(1+r)^n}{(1+r)^n - 1}\right\}$$
where N is the amount borrowed, r is the interest rate per period expressed as a decimal and n is the number of interest periods.
3. The total cost of a loan can be calculated by multiplying the amount of each repayment by the number of repayments to be made.
4. The length of time that it will take to repay a loan can be calculated by using guess and refine methods.

EXERCISE 8D

Loan repayments

For questions **1** to **3** use the formula, $M = N\left\{\frac{r(1+r)^n}{(1+r)^n - 1}\right\}$.

1 Yiannis takes out a \$10 000 loan over 5 years at 10% p.a. reducible interest with five equal annual repayments to be made. Use the formula to calculate the amount of each annual repayment.

2 WE11 Use the formula to calculate the amount of each monthly repayment on a loan of \$8000 to be repaid over 4 years at 12% p.a.

3 Use the formula to calculate the amount of each monthly repayment on each of the following loans.

a \$2000 at 12% p.a. over 2 years
b \$15 000 at 9% p.a. over 5 years
c \$120 000 at 6% p.a. over 20 years
d \$23 000 at 9.6% p.a. over 5 years
e \$210 000 at 7.2% p.a. over 25 years

4 Javier and Diane take out a \$175 000 home loan. If the interest rate on the loan is 8.4% p.a. reducible and the term of the loan is 25 years, calculate the amount of each monthly repayment.

5 Jiro purchases a computer on terms. The cash price of the computer is \$3750. The terms are a deposit of 10% with the balance paid in equal monthly instalments at 9% p.a. reducible interest over 3 years.

a Calculate Jiro's deposit on the computer.
b What is the balance owing on the computer?
c Calculate the amount of each monthly repayment.

6 Jeremy and Patricia spend $15 000 on new furnishings for their home. They pay a 15% deposit on the furnishings with the balance paid in equal monthly instalments at 18% p.a. interest over 4 years. Calculate the amount of each monthly repayment.

7 Thanh is purchasing a car on terms. The cash price of the car is $35 000 and he pays a $7000 deposit.
- **a** What is the balance owing on the car?
- **b** If the car is to be repaid in equal weekly instalments over 5 years at an interest rate of 10.4% p.a. reducible interest, calculate the amount of each weekly payment.

8 WE12 Ron borrows $13 500 to purchase a car. The loan is to be repaid in equal monthly instalments over a 3-year term at an interest rate of 15% p.a. Calculate the total repayments made on the loan.

9 Calculate the total repayments on each of the following loans.
- **a** $4000 at 8.4% p.a. reducible interest to be repaid over 2 years in equal monthly repayments
- **b** $20 000 at 13.2% p.a. reducible interest to be repaid over 6 years in equal monthly instalments
- **c** $60 000 at 7.2% p.a. reducible interest to be repaid over 15 years in equal monthly instalments
- **d** $150 000 at 10.8% p.a. reducible interest to be repaid over 20 years in equal monthly instalments

10 MC A loan of $5000 is taken out at 9% p.a. reducible interest over 4 years. Which of the following will give the amount of each monthly repayment?

A $M = 5000 \times \left\{ \dfrac{0.09 \times 1.09^4}{1.09^4 - 1} \right\}$

B $M = 5000 \times \left\{ \dfrac{0.09 \times 1.09^{48}}{1.09^{48} - 1} \right\}$

C $M = 5000 \times \left\{ \dfrac{0.0075 \times 1.0075^4}{1.0075^4 - 1} \right\}$

D $M = 5000 \times \left\{ \dfrac{0.0075 \times 1.0075^{48}}{1.0075^{48} - 1} \right\}$

11 MC A loan of $12 000 is taken out at 12% p.a. reducible interest in equal monthly instalments over 5 years. The total amount of interest paid on the loan is:

A $266.93 **B** $4015.80
C $7200 **D** $16 015.80

12 WE13 A loan of $75 000 is taken out over 15 years at 9% p.a. reducible interest. The minimum monthly repayment is $760.70. Calculate how long it will take to repay the loan at $1000 per month.

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13 A $150 000 loan is taken out over a 25-year term. The interest rate is 9.6% p.a.
- **a** Calculate the minimum monthly repayment.
- **b** Calculate the total repayments on the loan.
- **c** Calculate the length of time that it will take to repay the loan at $1600 per month.
- **d** Calculate the total saving on the loan by repaying the loan at $1600 per month.

Further development

14 Link borrows $48 000, taken out over 10 years. The loan is to be repaid in monthly instalments. Calculate the amount of each monthly repayment at each of the following interest rates.
- **a** 6% p.a. **b** 7% p.a.
- **c** 9% p.a. **d** 12 % p.a.

15 A loan of $200 000 over 20 years has interest charged monthly at a rate of 9% p.a.
- **a** Calculate the monthly repayment.
- **b** After 3 years the balance owing is $187 676.80. The interest rate then rises to 10%. What will be the new repayment required to pay the loan off in the remaining 17 years?

16 Ben took out of loan for $20 000 to buy a new car. The contract required that he repay the loan over 5 years with monthly instalments of $405.53. Use a trial-and-error method together with the loan repayment formula to find the interest rate that Ben is paying.

17 Tim has borrowed $245 000 to buy a house. He agrees to repay the reducing balance loan over 15 years with monthly instalments at 9.3% p.a. (adjusted monthly). Find:
- **a** the monthly repayment
- **b** the principal repaid and the interest paid during the first repayment.

18 Terry is repaying a $352 000 loan over 15 years with quarterly instalments at 6.25% p.a. (adjusted quarterly).
- **a** Calculate the amount of each quarterly repayment.
- **b** Calculate the total amount repaid on the loan.
- **c** Calculate the total interest paid on the loan.
- **d** Show whether the loan would have cost more or less, and by how much, had the repayments been made monthly and interest calculated monthly.

19 Stefanie borrowed $18 000 for a car. The reducing balance loan at 8% p.a. was for a term of 5 years and can be paid in annual, quarterly or monthly repayments. The interest will be calculated and added at the same time as each repayment. Find the cheapest way to repay the loan.

Types of loan arrangements

Research one example of each of the following types of loans.

A. Hire purchase agreement

This is the type of loan where a major item is purchased on terms. Usually a deposit is paid and then the balance plus interest is repaid over an agreed period of time.

B. Personal loan

This is a loan taken out from a bank or other financial institution. It can be used for any purpose and is *unsecured*. This means that there is no item of property that the bank can claim if repayments are not made.

C. Home loan

This is a secured loan, which means that, if the repayments are not made, the bank can claim the property and sell it to reclaim the amount outstanding on the loan.

For each of the above loans, answer the following questions.

1 What is the interest rate? Is interest calculated at a flat or reducible rate?

2 Over what term can the loan be repaid?

3 How regularly must repayments be made?

4 Can additional repayments be made to shorten the term of the loan?

5 Can the interest rate be altered after repayments have begun to be made?

6 What other fees and charges apply to borrowing the money?

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Annuities loans, graphs

Computer Application 4: Graphs of annuities and loans

Most financial institutions will provide graphs that show the growth of an annuity and the declining balance of a loan. These graphs can be obtained by either visiting the bank or by going to the internet site for the relevant financial institution.

Obtain a copy of a graph showing the growth of an investment and the declining balance of a loan.

Alternatively, develop a spreadsheet that shows the growth of an annuity and the declining balance of a loan and use the charting function of the spreadsheet to draw the graph.

Access the Word file 'Annuities, Loans, Graphs' from the *Maths Quest General Mathematics HSC Course* eBookPLUS.

SUMMARY

Future value of an annuity

- An annuity is where regular equal contributions are made to an investment. The interest on each contribution compounds as additions are made to the annuity.
- The future value of an annuity is the value that the annuity will have at the end of a fixed period of time.
- The future value of an annuity can be calculated using the formula:

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

where M is the contribution per period paid at the end of the period, r is the percentage interest rate per compounding period (expressed as a decimal) and n is the number of compounding periods.

- The amount of each contribution per period in an annuity can be found using the formula $M = \dfrac{Ar}{(1+r)^n - 1}$.

Present value of an annuity

- The present value of an annuity is the single sum that would need to be invested at the present time to give the same financial outcome at the end of the term.
- The present value of an annuity can be calculated using the formula:

$$N = \frac{A}{(1+r)^n}$$

where A is the future value of the annuity.

- An alternative formula to use is:

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

where M is the contribution made to the annuity per interest period.

Use of tables

- A table can be used to find the present or future value of an annuity.
- The table shows the present or future value of $1 under an annuity.
- The present or future value of $1 must be multiplied by the contribution per period to calculate its present or future value.

Loan repayments

- The present value of an annuity formula can be used to calculate the amount of each periodical repayment in a reducing balance loan. This is done by considering the present value of an annuity as the amount borrowed and making M the subject of the formula.
- The formula to be used is $M = N\left\{\dfrac{r(1+r)^n}{(1+r)^n - 1}\right\}$.
- The total amount to be repaid during a loan is calculated by multiplying the amount of each monthly repayment by the number of repayments to be made.

CHAPTER REVIEW

MULTIPLE CHOICE

1 Jenny invests $1000 per year for 20 years in an annuity. The interest rate is 6.5% p.a. and interest is compounded annually. The future value of the annuity is:

A $3523.65 **B** $11 018.51
C $18 825.31 **D** $38 825.31

2 Madeline invests $1000 per year for 20 years in an annuity. The interest rate is 6.5% p.a. and interest is compounded annually. The present value of the annuity is:

A $3523.65 **B** $11 018.51
C $18 825.31 **D** $38 825.31

3 Which of the following investments has the greatest future value after 10 years?

A An annuity of $500 per year at 7.75% p.a., with interest compounded annually
B An annuity of $250 per six months at 7.6% p.a., with interest compounded six-monthly
C An annuity of $125 per quarter at 7.2% p.a., with interest compounded quarterly
D A single investment of $3400 at 7.9% p.a., with interest compounded annually

4 A loan of $80 000 is taken out over a 20-year term at an interest rate of 9% p.a. The monthly repayment is $719.78. What would the total saving be if the term were reduced to 15 years?

A $91.63 **B** $16 493.40
C $21 991.20 **D** $26 693.40

SHORT ANSWER

1 Calculate the amount to which each of the following investments will grow.

a $3500 at 12% p.a. for 3 years, with interest compounded annually
b $2000 at 8% p.a. for 5 years, with interest compounded six-monthly
c $15 000 at 9.2% p.a. for 8 years, with interest compounded quarterly
d $4200 at 13.2% p.a. for 2 years, with interest compounded monthly

2 $400 per year is invested into an annuity at 7% p.a., with interest compounded annually. Use the formula $A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$ to calculate the value of the annuity after 20 years.

3 Use the formula $A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$ to calculate the future value of each of the following annuities.

a $500 invested per year for 25 years at 12% p.a., with interest compounded annually
b $1000 invested every 6 months for 10 years at 9% p.a., with interest compounded six-monthly
c $600 invested every 3 months for 5 years at 7.2% p.a., with interest compounded quarterly
d $250 invested per month for 20 years at 12% p.a., with interest compounded monthly

4 An annuity consists of $100 deposits every month for 15 years. The interest rate is 9% p.a. and interest is compounded six-monthly. Find the future value of the annuity.

5 Use the formula $M = \frac{Ar}{(1+r)^n - 1}$ to calculate the amount of each annual contribution to an annuity to achieve a savings goal of $800 000 in 40 years at an interest rate of 8% p.a., with interest compounded annually.

6 Calculate the amount of each contribution to the following annuities.

a $50 000 in 10 years at 6% p.a., with interest compounded annually and annual deposits
b $250 000 in 30 years at 12% p.a., with interest compounded six-monthly and contributions made every six months
c $120 000 in 20 years at 16% p.a., with interest compounding quarterly and contributions made quarterly

7 Use the formula $N = \frac{A}{(1+r)^n}$ to calculate the present value of an annuity if it is to have a future value of $350 000 in 30 years' time at an interest rate of 10% p.a., with interest compounded annually.

8 Calculate the present value of the following annuities with a future value of:

a $10 000 after 10 years at 5% p.a., with interest compounded annually
b $400 000 after 40 years at 12% p.a., with interest compounded annually
c $5000 after 5 years at 9% p.a., with interest compounded six-monthly
d $120 000 after 8 years at 15% p.a., with interest compounded quarterly.

9 Phuong wants to purchase a car in 3 years. He feels that he will need $15 000. The best investment he can find is at 8.5% p.a., interest compounded quarterly. What is the present value of this investment?

10 Gayle invests $400 per year in an annuity. The investment is at 6% p.a., with interest compounded annually. Gayle plans to invest in the annuity for 25 years. Use the formula $N = M\left\{\dfrac{(1+r)^n - 1}{r(1+r)^n}\right\}$ to calculate the present value of this annuity.

11 When Joanne begins work at 18, she invests $100 per month in a retirement fund. The investment is at 9% p.a., with interest compounded six-monthly.

- **a** If Joanne is to retire at 60 years of age, what is the future value of her annuity?
- **b** What is the present value of this annuity?

12 Use the table of future values of $1 on page 261 to calculate the future value of an annuity of $4000 deposited per year at 7% p.a. for 8 years, with interest compounded annually.

13 Use the table of future values of $1 to calculate the future value of the following annuities.

- **a** $750 invested per year for 5 years at 8% p.a., with interest compounded annually
- **b** $3500 invested every six months for 4 years at 12% p.a., with interest compounded six-monthly
- **c** $200 invested every 3 months for 2 years at 16% p.a., with interest compounded quarterly
- **d** $1250 invested every month for 3 years at 10% p.a., with interest compounded six-monthly

14 Use the table of present values of $1 on pages 262–3 to calculate the present value of an annuity of $500 invested per year for 6 years at 9% p.a., with interest compounded annually.

15 Use the table of present values to calculate the present value of each of the following annuities.

- **a** $400 invested per year for 5 years at 10% p.a., with interest compounded annually
- **b** $2000 invested every six months for 5 years at 14% p.a., with interest compounded six-monthly
- **c** $500 invested every three months for $2\frac{1}{2}$ years at 16% p.a., with interest compounded quarterly
- **d** $300 invested every month for 4 years at 12% p.a., with interest compounded half-yearly

16 Use the formula $M = N\left\{\dfrac{r(1+r)^n}{(1+r)^n - 1}\right\}$ to calculate the amount of each monthly repayment on a loan of $28 000 to be repaid over 6 years at 12% p.a.

17 Scott borrows $22 000 to purchase a car. The loan is taken out over a 4-year term at an interest rate of 9.6% p.a., with the loan to be repaid in equal monthly repayments.

- **a** Calculate the amount of each monthly repayment.
- **b** Calculate the total amount that is repaid on the loan.

18 Calculate the total repayments made on a home loan of $210 000 to be repaid in equal monthly repayments over 25 years at an interest rate of 8.4% p.a.

19 Adam buys a new lounge suite for $4400 and pays for it on his credit card. The interest rate on the credit card is 21% p.a. Adam hopes to pay the credit card off in two years by making equal monthly repayments.

- **a** Calculate the amount of each monthly repayment that Adam should make.
- **b** Calculate the total amount that Adam will make in repayments.
- **c** Calculate the amount of interest that Adam will pay.

EXTENDED RESPONSE

1 Lien invests $2000 per year in an annuity. The term of the annuity is 20 years and the interest rate is 8% p.a., with interest compounding annually.

- **a** Calculate the future value of this annuity.
- **b** Calculate the present value of this annuity.

c By how much will the future value of the annuity increase if Lien deposits $500 per quarter and interest is compounded quarterly?

2 Eddie has the goal of saving $1 000 000 over his working life, which he expects to be 40 years. Over the period of his working life, Eddie expects to be able to obtain an average 7% p.a. in interest with interest compounded every six months.

a Calculate the present value of this annuity.

b Use the formula $M = \dfrac{Ar}{(1+r)^n - 1}$ to calculate the amount of each six-monthly contribution to the annuity.

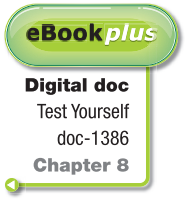
c For the first 10 years of the annuity Eddie makes no contributions, preferring to direct all his money into paying off a mortgage. At that time he makes a single contribution to catch up on the annuity. What amount must Eddie deposit?

3 Jim and Catherine take out a $150 000 loan. The interest rate on the loan is 12% p.a. and the loan is to be repaid in equal monthly repayments over a 20-year term.

a Use the formula $M = N\left\{\dfrac{r(1+r)^n}{(1+r)^n - 1}\right\}$ to calculate the amount of each monthly repayment.

b Calculate the total amount of interest that Jim and Catherine will need pay on this loan.

c Calculate the saving that Jim and Catherine will make by repaying the loan over a 12-year term.



eBookplus ACTIVITIES

Are you ready?

Digital docs *(page 248)*
- SkillSHEET 8.1 (doc-1379): Finding values of n and r in financial formulas.
- SkillSHEET 8.2 (doc-1380): Calculating simple interest.
- SkillSHEET 8.3 (doc-1381): Calculating compound interest.
- SkillSHEET 8.4 (doc-1383): Reading financial tables.

8A Future value of an annuity

Tutorial
- WE2 int-2429: Perform a calculation of annuity. *(page 250)*

Digital docs
- SkillSHEET 8.1 (doc-1379): Finding values of n and r in financial formulas. *(page 252)*
- SkillSHEET 8.2 (doc-1380): Calculating simple interest. *(page 252)*
- SkillSHEET 8.3 (doc-1381): Calculating compound interest. *(page 252)*
- Spreadsheet (doc-1382): Annuity calculator. *(page 255)*

8B Present value of an annuity

Tutorial
- WE8 int-2430: Perform a calculation of annuity. *(page 257)*

Digital doc
- WorkSHEET 8.1 (doc-1383): Apply your knowledge of annuities. *(page 259)*

8C Future and present value tables

Digital doc
- SkillSHEET 8.4 (doc-1383): Reading financial tables. *(page 263)*

8D Loan repayments

Tutorial
- WE13 int-2431: Perform a calculation of loan repayments. *(page 266)*

Digital docs
- WorkSHEET 8.2 (doc-1385): Apply your knowledge of loan repayments. *(page 268)*
- Word (doc-3330): Annuities, loans, graphs. *(page 269)*

Chapter review

- Test Yourself (doc-1386): Take the end-of-chapter test to test your progress. *(page 273)*

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